1. In this problem we will show that $\lim_{x\to 0}\sin(x) = 0$.

   (a) Use $\lim_{x\to 0+}\frac{\sin x}{x} = 1$ to show that there exist $\delta > 0$ such that if $0 < x < \delta$, then
   $$0 < \sin(x) < 2x$$

   (b) Use $\lim_{x\to 0-}\frac{\sin x}{x} = 1$ to show that there exist $\delta > 0$ such that if $-\delta < x < 0$, then
   $$2x < \sin(x) < 0$$

   (c) Combine (a) and (b) to prove $\lim_{x\to 0}\sin(x) = 0$.

2. Show that $\lim_{x\to 0}\cos x = 1$ by using $\cos x = 1 - 2\sin^2(\frac{x}{2})$

3. Use the following

   - limit laws
   $$\lim_{x\to a}(f(x) \pm g(x)) = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x),$$
   $$\lim_{x\to a}(f(x)g(x)) = \lim_{x\to a} f(x) \lim_{x\to a} g(x),$$

   - the fact that
   $$\lim_{x\to 0}\sin(x) = 0, \qquad \lim_{x\to 0}\cos(x) = 1,$$

   - the trigonometric laws
   $$\sin(x+h) = \sin(x)\cos(h) + \cos(x)\sin(h),$$
   $$\cos(x+h) = \cos(x)\cos(h) - \sin(x)\sin(h)$$

   to show that sine and cosine functions has the direct substitution propery
   $$\lim_{x\to a}\sin(x) = \sin(a),$$
   $$\lim_{x\to a}\cos(x) = \cos(a),$$

   (Hint: $\lim_{x\to a}\sin(x) = \lim_{h\to 0}\sin(a+h)$)